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APPLICATION OF THE METHOD OF LEAST SQUARES TO ENGINE-COOLING ANALYSIS

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SUMMARY

The flexibility of the NACA method of correlating engine-cooling data is shown in this report to be improved when the data are adjusted analytically to the correlation equation by the method of least squares. Engine-cooling data, to be correlated graphically, must be obtained from tests in which engine-charge-air flow and cooling-air flow are carefully controlled. The least-squares method is adapted to the correlation of engine-cooling data in which the flows of charge air and cooling air, if measured accurately, may be varied in any manner. The values of the correlation exponents determined by the least-squares method are unique and are not dependent upon the curve-fairing ability of the analyst.

Curve fitting by the method of least squares is discussed briefly and a solution is indicated for the values of the constants in an equation that can be identified with the engine-cooling-correlation equation. The NACA method of correlating engine-cooling data is illustrated by a graphical analysis of typical engine-cooling data. The same data are then correlated by the least-squares method. It is demonstrated that engine-cooling data not adapted to graphical correlation may be easily reduced by the least-squares method.

INTRODUCTION

The NACA method of correlating engine temperatures with the principal variables that determine engine temperatures has been developed for application as a graphical method (references 1, 2, and 3). An essential step in the graphical process is the evaluation of constant exponents in the correlation equation from logarithmic graphs of data for which either engine-charge-air flow or cooling-air flow was held constant for a series of tests. It is frequently difficult, especially in flight testing, to maintain perfect constancy of the engine-operation variables. Under this condition the data can be correlated graphically only after trial and error corrections have been applied. Such data can be correlated directly by the method of least squares with precision limited only by the accuracy of the data. When data can be obtained from very carefully controlled tests, a graphical correlation of the data can be performed more rapidly than a least-squares correlation and with equal precision. The least-squares method is recommended primarily for the correlation of engine data in which the engine-operation variables, although measured accurately, could not be held constant. A less important, though interesting, application of the least-squares method is its use as a supplement to the graphical correlation.

If the method of least squares is to be applied to the analysis of any data, it is necessary to know the form of the equation to which the data are to be fitted. The NACA method of correlating engine-cooling data employs the correlation equation in various forms. The present work identifies one form of the equation with a simple expression involving three variables and three unknown constants; the method of least squares is applied to determine the values of the constants in the equation. Inasmuch as the method of least squares does not require a systematic change in any of the variables, engine-cooling data involving simultaneous and irregular variation of engine-charge-air flow and cooling-air flow can be correlated analytically with precision.

The purpose of this report is to show that it is practical to apply a least-squares method to the correlation of engine-cooling data. The theory of least squares will be discussed briefly and a general solution will be obtained for the values of three unknown constants in an equation of three variables. Engine-cooling-correlation procedure will be described briefly and a graphical presentation of typical data will be supplemented by a least-squares correlation of the same data. The least-squares method will be used to correlate engine-cooling data which cannot readily be correlated by a graphical method.

ANALYSIS

Curve Fitting by the Method of Least Squares

The theory of least squares holds an important place in the mathematics of observations and is closely related to the laws of probability and the Gaussian law of error. A useful application in observational work is curve fitting by the method of least squares.

The term "curve fitting" applies to the determination of the values of the constants in an equation of assumed form such that the chosen equation is the best explicit representation of a given set of data. If a deviation is defined as the difference between a datum value of the dependent variable and the corresponding value on the fitted curve, a curve is regarded as representing the best fit to a given set of data when the sum of the squared deviations is a minimum. This condition also demands that the sum of the deviations be zero. The form of the equation is always an assumption whether it be admittedly empirical or undeniably based on physical laws.

The derivation of conventional formulas for curve fitting by the method of least squares is given in a number of textbooks; three such textbooks are listed as references 4, 5, and 6. A simplified derivation will be given herein for an equation of three variables and three unknown constants. The form of the equation chosen can be identified with the NACA cooling-correlation equation.

Data are given as a collection of coordinate values of the variables x, y, and z: (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n), where n is the number of values. Assume that the variable y can be represented explicitly in terms of the variables x and z by an equation of the form

$$y = ax + bz + c \tag{1}$$

where a, b, and c are constants. It is desired to determine such values of the constants a, b, and c that equation (1) will most closely fit the data. Represent any point by (x_i, y_i, z_i). If x and z are regarded as independent variables, the deviation δ_i of any datum value y_i from the locus of equation (1) is

$$\delta_i = y_i - y$$

$$\delta_i = y_i - ax_i - bz_i - c \tag{2}$$

According to the theory of least squares, equation (1) will best fit the data when the sum of the squared deviations is a minimum. Let the sum of the squared deviations be S; that is,

$$S = \sum_{i=1}^{i=n} \delta_i{}^2$$

Insofar as the coefficients a, b, and c may affect the value of S, this value will be a minimum when

$$\frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0, \quad \frac{\partial S}{\partial c} = 0$$

A squared deviation is

$$\delta_i{}^2 = (y_i - ax_i - bz_i - c)^2$$

$$\begin{aligned} \delta_i{}^2 = {} & y_i{}^2 + a^2x_i{}^2 + b^2z_i{}^2 + c^2 \\ & -2ax_iy_i - 2bz_iy_i - 2cy_i \\ & + 2abx_iz_i + 2bcz_i + 2acx_i \end{aligned}$$

Sum the squared deviations and drop the subscripts. Then,

$$\begin{aligned} S = \sum_{i=1}^{i=n} \delta_i{}^2 = {} & \sum y^2 + a^2\sum x^2 + b^2\sum z^2 + nc^2 \\ & - 2a\sum xy - 2b\sum zy - 2c\sum y \\ & + 2ab\sum xz + 2bc\sum z + 2ac\sum x \end{aligned}$$

The partial derivatives of S with respect to a, b, and c, respectively, are

$$\frac{\partial S}{\partial a} = 2a\sum x^2 + 2b\sum xz + 2c\sum x - 2\sum xy$$

$$\frac{\partial S}{\partial b} = 2a\sum xz + 2b\sum z^2 + 2c\sum z - 2\sum zy$$

$$\frac{\partial S}{\partial c} = 2a\sum x + 2b\sum z + 2nc - 2\sum y$$

The partial derivatives equated to zero yield three equations

$$\left.\begin{aligned}
\left(\sum x^2\right)a + \left(\sum xz\right)b + \left(\sum x\right)c &= \sum xy \\
\left(\sum xz\right)a + \left(\sum z^2\right)b + \left(\sum z\right)c &= \sum zy \\
\left(\sum x\right)a + \left(\sum z\right)b + nc &= \sum y
\end{aligned}\right\} \qquad (3)$$

The equations (3) solved simultaneously give the values of the constants a, b, and c that will make equation (1) best fit the data. A general solution for the values of a, b, and c, with a numerical example, is given in appendix A.

In order to obtain the values of the constants a, b, and c, either graphically or by the method of least squares, the experimental values of the original data must actually contain sufficient information for the purpose. It is desirable to have a large number of test points covering a wide range for all the variables. Occasionally, the simultaneous equations (3) are nearly exact multiples of each other, for which case the values of a, b, and c would be indeterminate. This result may be due to insufficient range of the data. The solution of equations (3) may sometimes appear inexact when determined by ratios of small differences between relatively large quantities. It must be remembered that the sums of the squares and products of the datum values are accurate to the same number of decimal places as the individual values; small differences between large sums are likewise accurate to that number of decimal places. The detail required in the calculations therefore makes desirable the use of a calculating machine in solving equations (3).

Advantages of Least-Squares Method

Curve fitting by the method of least squares has several advantages over a graphical method. One advantage is that the determined value of each constant is unique. Least-squares computations performed without error and with the proper precision can yield only that value of each constant which the data determine. The human element is not involved as is the case with graphical fairing.

A second advantage is the quantitative measure of the deviation of each datum value from the best determinable value. Using datum values of the independent variables x and z permits the corresponding deviation of y to be computed by use of equation (2). A small value for the sum of the deviations (ideally zero) indicates that the computations are probably free from error. Experimental values that have a deviation greater than the estimated experimental accuracy may be discarded. A repetition of the work then yields much more reliable values of the constants in equation (1). The squared deviations may also be tabulated and the standard deviation and a simple form of the probable error computed by equations (4) and (5), respectively, as

$$\text{Standard deviation} = \pm \sqrt{\frac{\sum \delta^2}{n}} \tag{4}$$

$$\text{Probable error} = \pm 0.67 \sqrt{\frac{\sum \delta^2}{n}} \tag{5}$$

The standard deviation represents a mean deviation for all the data. Probable error is the limit of error for one-half the experimental data.

APPLICATION OF METHOD OF LEAST SQUARES

The Engine-Cooling-Correlation Equation

The engine-cooling-correlation method was developed to coordinate engine temperatures with the principal variables that determine engine temperatures. A few tests made under carefully controlled operating conditions, easily attainable in a wind tunnel or on a test stand, serve to establish an engine-cooling correlation. This correlation may then be used to predict engine temperatures that result from specified operating conditions or to determine operating conditions requisite to maintain specified temperature limits.

In order to make cooling tests of an air-cooled engine for presentation by the correlation method, it is necessary to obtain, as basic data, measurements of the quantities listed in the following table of symbols:

T_h reference head temperature (average indication of all imbedded head thermocouples, or all rear-spark-plug gasket thermocouples), °F

T_a cooling-air temperature (stagnation-air temperature in front of engine), °F

T_e engine charge-air temperature ahead of carburetor, °F

σ_a cooling-air-density ratio based on stagnation density in front of engine

Δp cooling-air pressure drop across the engine, inches of water

W_e weight rate of charge-air flow (without fuel), pounds per second

N engine crankshaft speed, rpm

r blower gear ratio

d blower impeller diameter, feet

A complete list of symbols appears in appendix B.

The principles of engine-cooling correlation and the development of the technique of applying these principles are set forth in references 1, 2, and 3. A general statement of the correlation principle is that the ratio of cooling-temperature differential to heating-temperature differential is a function of a relationship between internal flow of heating fluid and external flow of cooling fluid. This relationship is expressed by

$$\frac{T_h - T_a}{T_g - T_h} = c_1 \frac{W_e{}^a}{(\sigma_a \Delta p)^b} \qquad (6)$$

In equation (6), c_1 is a constant, a and b are constant exponents associated with W_e and $\sigma_a \Delta p$, respectively, and T_g is the mean effective gas temperature, which is defined in references 1, 2, and 3. The mean effective gas temperature is a hypothetical average temperature used in engine-analysis computations to replace the continuously varying actual temperature of the charge and combustion products within the engine cylinder. A procedure for computing the value of the mean effective gas temperature is given in reference 7 for the Pratt & Whitney R-2800 engine. Equation (6) is an engine-cooling-correlation equation based on cooling-air pressure drop. For simplicity, only this form of the equation will be used in the present report.

Graphical Correlation

The value of the exponent a can be determined graphically by plotting on logarithmic coordinates the ratio of the temperature differentials against weight flow of charge air for tests in which the fuel-air ratio and sea-level cooling-air pressure drop $\sigma_a \Delta p$ are held constant. A plot of this type is shown in figure 1 (data from table I, test 241). Test numbers used herein are taken from reference 7, from which the data were obtained. The slope of the line, 0.565, is the value of the exponent a. A similar plot of the ratio of temperature differentials against sea-level cooling-air pressure drop $\sigma_a \Delta p$ yields the value of the exponent b. For the determination of the exponent b, tests must be made with the weight flow of engine charge air held constant and the fuel-air ratio held at the same constant value as that used in the tests to determine the exponent a. A plot of the type used to determine the value of b is shown in figure 2 (data from table I, test 240). The slope of the curve, -0.321, is the negative value of the exponent b.

The engine-cooling correlation (equation (6)) corresponding to the data presented in figures 1 and 2 can now be written as

$$\frac{T_h - T_a}{T_g - T_h} = c_1\left(\frac{W_e^{a/b}}{\sigma_a \Delta p}\right)^b = c_1\left(\frac{W_e^{1.76}}{\sigma_a \Delta p}\right)^{0.321} \tag{7}$$

The value of the constant c_1 can be established with the same data as that used to find a and b by plotting the ratio of temperature differentials against the relation $\left(W_e^{a/b}/\sigma_a \Delta p\right)$. Such a plot is presented in figure 3, which is the graphically established engine-cooling correlation. The value of the constant $c_1 = 0.560$ was computed from coordinate values read from the faired curve. The graphically determined correlation is given by

$$\frac{T_h - T_a}{T_g - T_h} = 0.560\left(\frac{W_e^{1.76}}{\sigma_a \Delta p}\right)^{0.321} \tag{8}$$

In order to use the correlation curve or equation it is necessary to know the variation of reference mean effective gas temperature $T_{g_{80}}$ with the fuel-air ratio; a typical plot is shown in figure 4. The subscript 80 indicates that the values of mean effective gas temperature were determined when the charge-air temperature T_e was 80° F. The curve in figure 4 was established (reference 7) by use of equation (8) with the data from table II, tests 242 and 244. Mean effective gas temperature corrected for carburetor-air temperature and blower-temperature rise can be computed from equation (9) and figure 4:

$$T_g = T_{g_{80}} + 0.8\left[T_e - 80 + \frac{r^2 d^2}{2.19}\,(N/1000)^2\right] \tag{9}$$

The derivation of equation (9) is given in reference 7.

Least-Squares Correlation

Two weaknesses exist in the graphical correlation procedure just described. One weakness is the necessity of maintaining a constant value of cooling-air pressure drop for one series of runs and a constant value of charge-air flow for another series of runs. To hold experimental values perfectly constant is not possible. A greater source of uncertainty is that, in fairing the construction curves (figs. 1 and 2), evaluation of the exponents a and b depends upon the discretion of the analyst. The use of least squares removes both of these difficulties. Equation (6) expressed logarithmically gives

$$\log\left(\frac{T_h - T_a}{T_g - T_h}\right) = a \log W_e + b \log (\sigma_a \Delta p) + \log c_1 \qquad (10)$$

In equation (10) the signs of the constants have been deliberately ignored because determination of the signs of the constants is of necessity performed in solving for their values.

The following identities should be made of values in equations (1) and (10):

$$y \equiv \log\left(\frac{T_h - T_a}{T_g - T_h}\right)$$

$$x \equiv \log W_e$$

$$z \equiv \log (\sigma_a \Delta p)$$

$$a \equiv a$$

$$b \equiv b$$

$$c \equiv \log c_1$$

Equation (10) can now be written in the form of equation (1):

$$y = ax + bz + c$$

and the values of the constants a, b, and c determined by the simultaneous solution of equations (3).

Inasmuch as there are three unknown constants in equation (1) (or (10)), at least three test points must be known in order to solve for the values of the constants. If only three points are known these may be substituted directly in equation (1) (or (10)) and the resulting three equations solved simultaneously. The values of the constants a, b, and c so determined will yield an equation satisfied by each of the three test points; there can be no deviation of a point from the curve. Unless the data are very accurate, the final equation may be greatly in error. The use of only three test points to determine the values of the three constants is the limiting condition for application of the least-squares correlation; when the number of datum values is increased, the reliability of the final equation is improved.

The most laborious computations involved in the application of least squares are those by which the sums of the variables and their cross products are obtained. These computations can be simplified by the use of forms for systematic tabulation similar to table III. The first three columns of table III contain datum values of the correlation variables taken from table I. The logarithms of the variables are listed in columns 6, 8, and 10, respectively. In the remaining columns, through 11, there are listed the cross products and squares of the quantities in columns 6, 8, and 10. The order of tabulation was chosen for convenience in making these computations. The sums of the squares and cross products obtained in table III have been used to set up equations (11) of which equations (3) are the type:

$$\left.\begin{aligned} 1.97933a + 7.90275b + 6.12224c &= -2.95038 \\ 7.90275a + 33.24726b + 25.50250c &= -12.58602 \\ 6.12224a + 25.50250b + 20c &= -9.74261 \end{aligned}\right\} \qquad (11)$$

The simultaneous solution of equations (11) yields the following values for the constants: (See appendix A.)

$$a = 0.578$$

$$b = -0.300$$

$$c = -0.281$$

The accuracy of the computations and the precision of the correlation have been evaluated by the computations performed in columns 12 to 16 of table III. The individual datum values x have been multiplied by the determined value of a and the products ax listed in column 12; similarly, in column 13 are tabulated the products bz. For each individual datum value, then, the sum of columns 12 and 13 and the constant c is tabulated in column 14, identified by

$$f(x, z) = ax + bz + c \qquad (12)$$

Points established by equation (12) lie on the curve of best fit; hence, the deviation of a datum value y of the dependent variable is determined by its difference from the point of best fit.

$$\delta = y - f(x,z)$$

The deviations of the datum values of y from the curve of best fit are tabulated in column 15, and the squared deviations are tabulated in column 16. The very small value of the sum of the deviations indicates that the work is probably free from computational error. The sum of the squared deviations has been used to find the standard deviation and probable error, equations (4) and (5).

The values of the constants a, b, and c determined by the work in table III were not regarded as final. A study of column 15 showed that the deviations of five of the test points (runs 7, 8, 9, 13, and 15) were considerably larger than the other deviations. These five points were eliminated from the array of data and a redetermination of the values of the constants was performed in table IV. These values, which are regarded as more reliable than those of table III, are shown in the following list, which is arranged for a quick comparison with the values obtained in table III and the values obtained by the graphical method:

	Graphical method	First least-squares correlation (Table III)	Final least-squares correlation (Table IV)
	$n = 20$	$n = 20$	$n = 15$
Exponent a	0.565	0.578	0.576
Exponent b	0.321	0.300	0.304
Exponent a/b	1.76	1.92	1.89
Constant c		-0.281	-0.276
Constant c_1	0.560	0.523	0.529
Standard deviation	±0.0099	±0.0089	±0.0051
Probable error	±0.0066	±0.0060	±0.0034

The standard deviation and probable error shown in the preceding table for the graphical method were obtained by using the deviation of datum values from the logarithmic form of equation (8). The standard deviation is a measure of the mean scatter of test points from the fitted curve. Using standard deviation as a basis for comparison, the final least-squares correlation (table IV) yields the equation of these three equations that best fits the data. The engine-cooling-correlation equations (13) and (14) obtained by the least-squares method are directly comparable with equation (8) obtained graphically:

First least-squares correlation (table III)

$$\frac{T_h - T_a}{T_g - T_h} = 0.523 \left(\frac{W_e^{1.92}}{\sigma_a \Delta p}\right)^{0.300} \tag{13}$$

Final least-squares correlation (table IV)

$$\frac{T_h - T_a}{T_g - T_h} = 0.529 \left(\frac{W_e^{1.89}}{\sigma_a \Delta p}\right)^{0.304} \tag{14}$$

In order to show how well these equations fit the data by which they have been established, equation (13) is shown plotted in figure 5 and equation (14), in figure 6. The first least-squares curve (fig. 5), which was established by the same data as were used for the graphical method, is directly comparable with the graphical curve (fig. 3). The final least-squares curve, figure 6, is established by select data (table IV) and is regarded as a close approach to the best possible adjustment.

A main purpose in applying the least-squares method to the correlation of engine-cooling data is to provide an exact and systematic means for finding the values of the constants in the engine-cooling-correlation equation. Precise correlation may not be very important as regards temperature prediction but in engine analysis every effort should be made to obtain the highest possible precision. Two different correlations have been established with the data listed in table I: the graphical correlation (equation (8)) and the final least-squares correlation (equation (14)). A comparison is shown in figure 7 of the average cylinder-head temperatures for two different operating conditions calculated by equations (8) and (14) as functions of cooling-air pressure drop. The reasonably close agreement between the temperatures predicted by the two equations shows that for temperature prediction a precise correlation is not necessary. On the other hand, the agreement between the values of the exponents obtained by the first and final least-squares correlations (tables III and IV) shows the exactness of the least-squares method. The fact that both least-squares correlations yielded equations from which the standard deviation of the data was less than that for the graphical correlation indicates the greater precision of the least-squares method.

THE CORRELATION OF MISCELLANEOUS ENGINE-COOLING DATA

In the comparison of the least-squares method of correlating engine-cooling data with the graphical method it was stated that the practice of making one series of tests with constant cooling-air pressure drop and another with constant engine-charge-air flow was not essential if the data were to be correlated analytically. In order to demonstrate this fact, special engine-cooling-correlation tests were made and only the fuel-air ratio was held constant (approximately 0.08); the engine speed, charge-air flow, and cooling-air pressure drop were deliberately varied from test to test. The data obtained during these special tests and the engine-cooling-correlation computations are presented in table V. A brief study of table V will show that the data are not suited to graphical analysis. A least-squares correlation of all the data of table V (17 test points) is performed in table VI. This correlation showed three of the test points (runs 15, 17, and 18) to have rather large deviations. These three test points were omitted from the array, and a final least-squares correlation was performed in table VII. The values obtained for the constants were: $a = 0.563$, $b = -0.305$, and $c = \log c_1 = -0.271$; the corresponding engine-cooling correlation, expressed by equation (15), is plotted in figure 8:

$$\frac{T_h - T_a}{T_g - T_h} = 0.535\left(\frac{W_e^{1.85}}{\sigma_a \Delta p}\right)^{0.305} \tag{15}$$

The differences between the correlations, equations (14) and (15), may be due to changes in the engine cowling made between test 241 and test 363 and also to differences in fuel. In tests 240, 241, 242, and 244, 100-octane blue aviation gasoline was used, whereas in test 363 the fuel was 100-octane green aviation gasoline containing aromatic compounds. The differences between equations (14) and (15) are actually within the experimental accuracy of the engine test data. Equation (15) and figure 8 show that engine-cooling data not adaptable to graphical analysis may be readily correlated by the least-squares method.

If miscellaneous engine-cooling data in which there was no systematic variation of charge- and cooling-air flows and for which the fuel-air ratio was not held constant are available, an approximate correlation can be obtained by use of the least-squares method and an assumed variation of mean gas temperature with fuel-air ratio. The reference mean effective gas temperature $T_{g_{80}}$ of the charge and combustion products in an engine cylinder is a physical characteristic of the fuel-air mixture and should be more or less the same for engines of a given type. This fact is borne out by similarity of the variation of mean effective gas temperature with fuel-air ratio for various air-cooled engines (references 1, 2, 3, and 7). Only very small error should result from the use of the gas-temperature curve, figure 4, with data obtained from any air-cooled engine. Use of this curve makes possible the evaluation of the ratio of temperature differentials. The subsequent correlation of the data may be performed graphically, if the data are suitable, or by the least-squares method in any case. An approximate correlation of miscellaneous cooling data obtained by use of an assumed gas-temperature curve should be useful at least for predicting engine temperatures.

A danger in using the least-squares procedure is in the temptation to attribute greater accuracy to the constants of the correlation equation computed by this process than is warranted by the accuracy of the data. There is no method or procedure for handling data that obviates the necessity for good judgment on the part of the analyst.

CONCLUSIONS

The application of the method of least squares to supplement the use of the NACA engine-cooling-correlation equation leads to the following conclusions:

1. Engine-cooling data, including data not adaptable to graphical analysis, can be correlated with precision by the method of least squares.

2. The values of the constants in the correlation equation determined by the method of least squares are unique and are not dependent upon the curve-fairing ability of the analyst.

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APPENDIX A

GENERAL SOLUTION FOR THREE SIMULTANEOUS EQUATIONS

The constants a, b, and c of equation (1) may be evaluated by the simultaneous solution of equations (3), namely:

$$\left.\begin{aligned}\left(\sum x^2\right)a + \left(\sum xz\right)b + \left(\sum x\right)c &= \sum xy\\ \left(\sum xz\right)a + \left(\sum z^2\right)b + \left(\sum z\right)c &= \sum zy\\ \left(\sum x\right)a + \left(\sum z\right)b + nc &= \sum y\end{aligned}\right\} \qquad (3)$$

The summations indicated in each of these equations may be identified as follows:

$$\sum x^2 = A = 1.97933$$

$$\sum xz = B = 7.90275$$

$$\sum x = C = 6.12224$$

$$\sum z = D = 25.50250$$

$$\sum z^2 = E = 33.24726$$

$$\sum xy = F = -2.95038$$

$$\sum yz = G = -12.58602$$

$$\sum y = H = -9.74261$$

where the numerical values are obtained from table III (or equations (11)), in which $n = 20$. If determinants are used to solve equations (3), the minors involved in the process may be identified as follows:

$$M_1 = nE - D^2 = 14.56769$$

$$M_2 = CD - nB = -1.92257$$

$$M_3 = BD - CE = -2.00783$$

$$M_4 = nA - C^2 = 2.10478$$

$$M_5 = BC - AD = -2.09533$$

$$M_6 = AE - B^2 = 3.35384$$

The four determinants necessary to the solution may be evaluated by use of the minors and summation identities:

$$\Delta_1 = AM_1 + BM_2 + CM_3 = 1.34826$$

$$\Delta_2 = FM_1 + GM_2 + HM_3 = 0.77878$$

$$\Delta_3 = FM_2 + GM_4 + HM_5 = -0.40451$$

$$\Delta_4 = FM_3 + GM_5 + HM_6 = -0.37943$$

The constants may be evaluated by the following ratios:

$$a = \frac{\Delta_2}{\Delta_1} = 0.57762$$

$$b = \frac{\Delta_3}{\Delta_1} = -0.30002$$

$$c = \frac{\Delta_4}{\Delta_1} = -0.28142$$

The numerical work performed in this appendix has been carried to five decimal places to maintain computational precision. Because the original data were accurate to only three significant figures, only three significant figures are retained in the final answer. The values used are therefore

$$a = 0.578$$

$$b = -0.300$$

$$c = -0.281$$

APPENDIX B

SYMBOLS

N	engine crankshaft speed, rpm
T_a	cooling-air temperature (stagnation-air temperature in front of engine), °F
T_e	engine charge-air temperature ahead of carburetor, °F
T_g	mean effective gas temperature, °F
ΔT_g	increment of mean effective gas temperature (See reference 7)
$T_{g_{80}}$	reference mean effective gas temperature (for 80° F charge-air temperature), °F
T_h	reference head temperature (average indication of all imbedded head thermocouples, or all rear-spark-plug gasket thermocouples), °F
W_e	weight rate of charge-air flow (without fuel), lb/sec
a,b,c,c_1	constants
d	blower impeller diameter, ft
n	number of test points
Δp	cooling-air pressure drop across the engine, in. water
r	blower gear ratio
δ	deviation of a datum value from the fitted curve
σ_a	cooling-air-density ratio based on stagnation density in front of engine

$c \equiv \log c_1$

$x \equiv \log W_e$

$$y \equiv \log\left(\frac{T_h - T_a}{T_g - T_h}\right)$$

$$z \equiv \log \sigma_a \Delta p$$

REFERENCES

1. Pinkel, Benjamin: Heat-Transfer Processes in Air-Cooled Engine Cylinders. NACA Rep. No. 612, 1938.

2. Schey, Oscar W., Pinkel, Benjamin, and Ellerbrock, Herman H., Jr.: Correction of Temperatures of Air-Cooled Engine Cylinders for Variation in Engine and Cooling Conditions. NACA Rep. No. 645, 1938.

3. Pinkel, Benjamin, and Ellerbrock, Herman H., Jr.: Correlation of Cooling Data from an Air-Cooled Cylinder and Several Multicylinder Engines. NACA Rep. No. 683, 1940.

4. Weld, LeRoy D.: Theory of Errors and Least Squares. The Macmillan Co., 1937.

5. Sokolnikoff, Ivan S., and Sokolnikoff, Elizabeth S.: Higher Mathematics for Engineers and Physicists. McGraw-Hill Book Co., Inc., 1934.

6. Gavett, G. Irving: A First Course in Statistical Method. McGraw-Hill Book Co., Inc., 1925.

7. Corson, Blake W., Jr., and McLellan, Charles H.: Cooling Characteristics of a Pratt & Whitney R-2800 Engine Installed in an NACA Short-Nose High-Inlet-Velocity Cowling. NACA ACR No. L4F06, 1944.

TABLE I

COOLING DATA AND CORRELATION COMPUTATIONS

[P. & W. R-2800 B-series engine, low blower, imbedded thermocouples, nonaromatic fuel. Data from reference 7.]

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Test	Run	Brake horsepower	N (rpm)	Charge-air flow (lb/hr)	Fuel flow (lb/hr)	Fuel-air ratio	$T_{g_{80}}$ (°F)	Carburetor temperature (°F)	ΔT_g (°F)	$\sigma_a \Delta p$ (in. water)	T_a (°F)	T_h (°F)	T_g (°F)	$\frac{T_h - T_a}{T_g - T_h}$	W_e (lb/sec)	$\frac{W_e^{1.76}}{\sigma_a \Delta p}$	$W_e^{1.92}$	$\frac{W_e^{1.92}}{\sigma_a \Delta p}$	$W_e^{1.89}$	$\frac{W_e^{1.89}}{\sigma_a \Delta p}$
240	1	1100	2120	8040	640	0.0796	1154	68	69	43.0	96	331	1223	0.264	2.234	0.096	4.68	0.109	4.57	0.106
	2	1100	2120	7973	646	.0810	1141	70	71	36.9	97	339	1212	.277	2.215	.110	4.60	.125	4.50	.122
	3	1100	2120	7987	640	.0802	1148	71	72	31.1	99	353	1220	.293	2.220	.131	4.62	.149	4.52	.145
	4	1100	2120	7937	630	.0794	1154	73	73	25.6	100	367	1227	.310	2.203	.157	4.56	.178	4.45	.174
	5	1100	2120	7803	630	.0808	1143	73	73	19.3	101	385	1216	.342	2.170	.203	4.43	.230	4.33	.224
	6	1100	2120	7770	613	.0788	1160	77	77	13.1	100	408	1237	.372	2.160	.296	4.39	.335	4.29	.327
	7	1100	2120	7750	613	.0791	1157	78	77	9.7	97	437	1234	.427	2.153	.398	4.36	.449	-------	-------
	8	1100	2120	7790	615	.0790	1158	81	80	30.8	102	367	1238	.304	2.165	.126	4.41	.143	-------	-------
	9	1100	2120	7677	619	.0806	1145	80	79	14.9	99	403	1224	.370	2.131	.254	4.28	.287	-------	-------
	10	1100	2120	7830	623	.0795	1154	80	79	31.1	102	362	1233	.299	2.175	.126	4.45	.143	4.34	.140
	12	1100	2120	7855	613	.0780	1167	71	72	31.4	91	350	1239	.291	2.181	.126	4.47	.142	4.36	.139
	13	1100	2120	7743	592	.0765	1180	70	71	19.5	91	374	1251	.323	2.151	.197	4.36	.224	-------	-------
	14	1100	2120	7695	592	.0769	1176	67	69	14.8	92	397	1245	.360	2.138	.257	4.30	.291	4.20	.284
	15	1100	2120	7578	565	.0746	1198	70	71	9.5	87	421	1268	.394	2.105	.390	4.18	.440	-------	-------
	16	1100	2120	7708	603	.0782	1165	67	69	14.7	91	396	1234	.364	2.141	.260	4.32	.294	4.22	.287
241	1	600	2120	4647	347	.0746	1198	70	71	13.5	84	338	1268	.273	1.291	.116	1.63	.121	1.62	.120
	2	800	2120	5793	454	.0784	1163	69	70	14.5	85	356	1233	.309	1.609	.159	2.49	.172	2.46	.170
	3	990	2120	7013	551	.0785	1162	69	70	14.2	87	383	1232	.349	1.947	.228	3.60	.254	3.52	.248
	4	1200	2120	8300	644	.0775	1171	68	69	14.4	88	404	1240	.378	2.307	.303	4.98	.346	4.86	.338
	5	600	2120	4613	355	.0769	1176	70	71	14.2	86	337	1247	.276	1.282	.109	1.61	.113	1.60	.113

TABLE II

COOLING DATA AND COMPUTATION OF GAS TEMPERATURE

[P. & W. R-2800 B-series engine, low blower, imbedded thermocouples, nonaromatic fuel. Data from reference 7.]

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Test	Run	Brake horse-power	N (rpm)	Charge-air flow (lb/hr)	Fuel flow (lb/hr)	Fuel-air ratio	Carbu-retor temper-ature (°F)	ΔT_g (°F)	$\sigma_a \Delta p$ (in. water)	W_e (lb/sec)	$\frac{W_e^{1.76}}{\sigma_a \Delta p}$	$\frac{T_h - T_a}{T_g - T_h}$	T_h (°F)	T_a (°F)	T_g (°F)	$T_{g_{80}}$ (°F) (Fig. 4)
242	1	800	2120	5820	461	0.0801	79	78	14.7	1.617	0.159	0.309	362	97	1220	1142
	2	800	2120	5730	424	.0740	78	77	15.0	1.593	.151	.303	368	98	1260	1183
	3	800	2120	5780	394	.0681	80	79	14.8	1.606	.155	.307	374	99	1270	1191
	4	800	2120	5840	382	.0655	80	79	14.9	1.623	.157	.308	373	98	1265	1186
	5	800	2120	6133	373	.0608	80	79	14.8	1.704	.172	.318	366	98	1210	1131
	6	800	2120	6740	385	.0571	81	80	15.0	1.871	.201	.334	350	99	1101	1021
	7	800	2120	5770	432	.0748	81	80	14.9	1.603	.154	.307	371	98	1260	1180
	8	800	2120	5790	396	.0683	82	81	14.8	1.609	.156	.308	376	99	1275	1194
	9	800	2120	7370	390	.0530	78	77	14.9	2.049	.237	.352	334	99	1003	926
	10	800	2120	6187	374	.0603	77	77	15.1	1.720	.172	.318	359	96	1185	1108
	11	800	2120	5950	377	.0633	75	75	15.2	1.654	.159	.309	364	94	1236	1161
	12	800	2120	5727	405	.0707	72	73	15.1	1.592	.152	.305	364	92	1255	1182
244	1	1400	2120	10533	1139	.1081	56	60	15.1	2.926	.438	.429	337	79	940	880
	2	1400	2120	10260	1038	.1011	59	62	14.2	2.850	.444	.430	350	81	975	913
	3	1400	2120	9853	890	.0904	60	63	14.9	2.737	.395	.412	382	82	1111	1048
	4	1110	2120	7697	608	.0791	61	64	15.1	2.139	.252	.359	377	81	1200	1136
	5	1100	1749	7608	605	.0795	61	39	15.3	2.113	.244	.355	362	81	1155	1116
	6	1040	2501	7688	606	.0788	61	95	14.5	2.137	.263	.364	397	81	1266	1171
	7	1630	2400	13117	1490	.1136	61	86	13.9	3.644	.701	.499	356	83	904	818

TABLE III

FIRST LEAST-SQUARES CORRELATION OF ENGINE-COOLING

DATA FROM TABLE I

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test	Run	W_e (lb/sec)	$\frac{T_h - T_a}{T_g - T_h}$	$\sigma_a \Delta p$ (in. water)	xz	x^2	x	xy	y	yz	z	z^2	ax	bz	f(x,z)	δ	δ^2
240	1	2.234	0.264	43.0	0.57021	0.12186	0.34908	-0.20191	-0.57840	-0.94480	1.63347	2.66822	0.20163	-0.49001	-0.56981	-0.00859	0.00007
	2	2.215	.277	36.9	.54121	.11928	.34537	-.19255	-.55752	-.87365	1.56703	2.45558	.19949	-.47008	-.55202	-.00550	.00003
	3	2.220	.293	31.1	.51702	.11996	.34635	-.18465	-.53313	-.79584	1.49276	2.22833	.20005	-.44780	-.52918	-.00395	.00002
	4	2.203	.310	25.6	.48304	.11766	.34301	-.17447	-.50864	-.71629	1.40824	1.98314	.19812	-.42244	-.50575	-.00289	.00001
	5	2.170	.342	19.3	.43254	.11321	.33646	-.15678	-.46597	-.59903	1.28556	1.65266	.19434	-.38564	-.47273	.00676	.00005
	6	2.160	.372	13.1	.37367	.11186	.33445	-.14363	-.42946	-.47982	1.11727	1.24829	.19318	-.33516	-.42341	-.00605	.00004
	7	2.153	.427	9.7	.32863	.11092	.33304	-.12308	-.36957	-.36468	.98677	.97372	.19236	-.29601	-.38508	.01551	.00024
	8	2.165	.304	30.8	.49935	.11253	.33546	-.17348	-.51713	-.76977	1.48855	2.21578	.19376	-.44654	-.53421	.01708	.00029
	9	2.131	.370	14.9	.38549	.10796	.32858	-.14188	-.43180	-.50658	1.17319	1.37637	.18979	-.35193	-.44357	.01177	.00014
	10	2.175	.299	31.1	.50375	.11388	.33746	-.17694	-.52433	-.78270	1.49276	2.22833	.19492	-.44780	-.53431	.00998	.00010
	12	2.181	.291	31.4	.50695	.11469	.33866	-.18156	-.53611	-.80252	1.49693	2.24080	.19561	-.44905	-.53487	-.00124	0
	13	2.151	.323	19.5	.42912	.11065	.33264	-.16326	-.49080	-.63315	1.29003	1.66418	.19213	-.38698	-.47628	-.01452	.00021
	14	2.138	.360	14.8	.38620	.10891	.33001	-.14643	-.44370	-.51924	1.17026	1.36951	.19061	-.35105	-.44187	-.00183	0
	15	2.105	.394	9.5	.31605	.10449	.32325	-.13075	-.40450	-.39549	.97772	.95594	.18671	-.29330	-.38802	-.01648	.00027
	16	2.141	.364	14.7	.38594	.10931	.33062	-.14511	-.43890	-.51234	1.16732	1.36264	.19097	-.35017	-.44063	.00173	0
241	1	1.291	.273	13.5	.12539	.01231	.11093	-.06255	-.56384	-.63733	1.13033	1.27765	.06407	-.33908	-.55644	-.00740	.00005
	2	1.609	.309	14.5	.23989	.04267	.20656	-.10535	-.51004	-.59235	1.16137	1.34878	.11931	-.34839	-.51051	.00047	0
	3	1.947	.349	14.2	.33344	.08373	.28937	-.13229	-.45717	-.52679	1.15229	1.32777	.16714	-.34566	-.45995	.00278	.00001
	4	2.307	.378	14.4	.42054	.13181	.36305	-.15339	-.42251	-.48942	1.15836	1.34180	.20970	-.34748	-.41921	-.00330	.00001
	5	1.282	.276	14.2	.12432	.01164	.10789	-.06032	-.55909	-.64423	1.15229	1.32777	.06232	-.34566	-.56477	.00568	.00003
	n=20			Sum	7.90275	1.97933	6.12224	-2.95038	-9.74261	-12.58602	25.50250	33.24726			Sum	0.00001	0.00157

a = 0.578
b = -0.300
c = -0.281

Standard deviation = ±0.0089
Probable error = ±0.0060

TABLE IV

FINAL LEAST-SQUARES CORRELATION OF ENGINE-COOLING

DATA FROM TABLE I

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test	Run	W_e (lb/sec)	$\frac{T_h - T_a}{T_g - T_h}$	$\sigma_a \Delta p$ (in. water)	xz	x^2	x	xy	y	yz	z	z^2	ax	bz	f(x,z)	δ	$δ^2$
240	1	2.234	0.264	43.0	0.57021	0.12186	0.34908	-0.20191	-0.57840	-0.94480	1.63347	2.66822	0.20104	-0.49693	-0.57216	-0.00624	0.00004
	2	2.215	.277	36.9	.54121	.11928	.34537	-.19255	-.55752	-.87365	1.56703	2.45558	.19890	-.47672	-.55409	-.00343	.00001
	3	2.220	.293	31.1	.51702	.11996	.34635	-.18465	-.53313	-.79584	1.49276	2.22833	.19946	-.45413	-.53094	-.00219	0.
	4	2.203	.310	25.6	.48304	.11766	.34301	-.17447	-.50864	-.71629	1.40824	1.98314	.19754	-.42841	-.50714	-.00150	0
	5	2.170	.342	19.3	.43254	.11321	.33646	-.15678	-.46597	-.59903	1.28556	1.65266	.19377	-.39109	-.47359	.00762	.00006
	6	2.160	.372	13.1	.37367	.11186	.33445	-.14363	-.42946	-.47982	1.11727	1.24829	.19261	-.33990	-.42356	-.00590	.00003
	10	2.175	.299	31.1	.50375	.11388	.33746	-.17694	-.52433	-.78270	1.49276	2.22833	.19434	-.45413	-.53606	.01173	.00014
	12	2.181	.291	31.4	.50695	.11469	.33866	-.18156	-.53611	-.80252	1.49693	2.24080	.19503	-.45540	-.53664	.00053	0
	14	2.138	.360	14.8	.38620	.10891	.33011	-.14643	-.44370	-.51924	1.17026	1.36951	.19005	-.35602	-.44224	-.00146	0
	16	2.141	.364	14.7	.38594	.10931	.33062	-.14511	-.43890	-.51234	1.16732	1.36264	.19040	-.35512	-.44099	.00209	0
241	1	1.291	.273	13.5	.12539	.01231	.11093	-.06255	-.56384	-.63733	1.13033	1.27765	.06388	-.34387	-.55626	-.00758	.00006
	2	1.609	.309	14.5	.23989	.04267	.20656	-.10535	-.51004	-.59235	1.16137	1.34878	.11896	-.35331	-.51062	.00058	0
	3	1.947	.349	14.2	.33344	.08373	.28937	-.13229	-.45717	-.52679	1.15229	1.32777	.16665	-.35055	-.46017	.00300	.00001
	4	2.307	.378	14.4	.42054	.13181	.36305	-.15339	-.42251	-.48942	1.15836	1.34180	.20908	-.35240	-.41959	-.00292	.00001
	5	1.282	.276	14.2	.12432	.01164	.10789	-.06032	-.55909	-.64423	1.15229	1.32777	.06213	-.35055	-.56469	.00560	.00003
	n=15			Sum	5.94411	1.43278	4.46927	-2.21793	-7.52881	-9.91635	19.58624	26.06127			Sum	-0.00007	0.00039

a = 0.576
b = -0.304
c = -0.276

Standard deviation = ±0.0051
Probable error = ±0.0034

TABLE V

COOLING DATA AND CORRELATION COMPUTATIONS

[P. & W. R-2800 B-series engine, low blower, imbedded thermocouples, aromatic fuel. Unpublished data.]

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Test	Run	Brake horse-power	N (rpm)	Charge-air flow (lb/hr)	Fuel flow (lb/hr)	Fuel-air ratio	$T_{g_{80}}$ (°F)	Carbu-rator temper-ature (°F)	ΔT_g (°F)	$\sigma_a \Delta p$ (in. water)	T_a (°F)	T_h (°F)	T_g (°F)	$\frac{T_h - T_a}{T_g - T_h}$	W_e (lb/sec)	$\frac{W_e^{1.85}}{\sigma_a \Delta p}$
363	1	1190	2120	8010	632	0.0789	1158	59	62	11.0	79.0	403	1220	0.397	2.225	0.3991
	2	1020	2120	7000	560	.0800	1150	60	63	10.8	80.0	390	1213	.377	1.944	.3167
	3	860	2120	6010	466	.0775	1167	64	66	10.8	79.5	377	1233	.348	1.669	.2389
	4	670	2120	5020	379	.0755	1177	63	66	11.2	78.5	355	1243	.311	1.394	.1652
	5	1150	2290	8030	637	.0793	1154	59	75	12.4	80.0	400	1229	.386	2.231	.3556
	6	970	2290	7020	563	.0802	1148	60	76	21.6	83.0	354	1224	.311	1.950	.1670
	8	1090	2490	8040	642	.0799	1150	59	92	20.7	82.0	374	1242	.336	2.233	.2135
	9	1250	2490	9010	739	.0820	1130	60	93	20.6	83.0	386	1223	.362	2.503	.2655
	10	580	2490	5030	379	.0753	1177	62	95	16.0	77.5	336	1272	.276	1.397	.1162
	11	640	2290	5030	381	.0761	1175	60	76	7.6	71.5	378	1251	.351	1.397	.2447
	12	550	2700	5050	385	.0762	1174	54	108	13.1	64.5	343	1282	.297	1.403	.1427
	13	720	2700	6060	466	.0769	1170	54	108	21.8	67.0	330	1278	.277	1.683	.1202
	14	670	2120	5050	387	.0766	1171	54	58	22.3	66.0	298	1229	.249	1.403	.0839
	15	1180	2120	8070	642	.0796	1153	54	58	21.8	69.0	346	1211	.320	2.242	----
	16	1050	2700	8070	648	.0803	1147	53	107	32.5	72.0	340	1254	.293	2.242	.1369
	17	1280	2700	9580	779	.0813	1137	52	106	46.8	77.5	346	1243	.299	2.661	----
	18	1180	2120	8060	626	.0777	1165	54	58	21.9	71.0	350	1223	.320	2.239	----

TABLE VI

FIRST LEAST-SQUARES CORRELATION OF ENGINE-COOLING

DATA FROM TABLE V

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test	Run	W_e (lb/sec)	$\sigma_a \Delta p$ (in. water)	$\frac{T_h - T_a}{T_g - T_h}$	xz	x^2	x	xy	y	yz	z	z^2	ax	bz	f(x,z)	δ	δ^2
363	1	2.225	11.0	0.397	0.3617	0.1206	0.3473	-0.1393	-0.4012	-0.4178	1.0414	1.0845	0.1938	-0.3083	-0.3960	-0.0052	0.00003
	2	1.944	10.8	.377	.2983	.0833	.2887	-.1223	-.4237	-.4379	1.0334	1.0679	.1611	-.3059	-.4263	.0026	.00001
	3	1.669	10.8	.348	.2299	.0495	.2225	-.1020	-.4584	-.4737	1.0334	1.0679	.1242	-.3059	-.4632	.0048	.00002
	4	1.394	11.2	.311	.1514	.0208	.1443	-.0732	-.5072	-.5322	1.0492	1.1008	.0805	-.3106	-.5116	.0044	.00002
	5	2.231	12.4	.386	.3810	.1215	.3485	-.1441	-.4134	-.4520	1.0934	1.1955	.1945	-.3236	-.4106	-.0028	.00001
	6	1.950	20.6	.311	.3810	.0841	.2900	-.1471	-.5072	-.6664	1.3139	1.7263	.1618	-.3889	-.5086	.0014	.00000
	8	2.233	20.7	.336	.4592	.1217	.3489	-.1653	-.4737	-.6234	1.3160	1.7319	.1947	-.3895	-.4763	.0026	.00001
	9	2.503	20.6	.362	.5236	.1588	.3985	-.1759	-.4413	-.5798	1.3139	1.7263	.2224	-.3889	-.4480	.0067	.00004
	10	1.397	16.0	.276	.1748	.0211	.1452	-.0812	-.5591	-.6732	1.2041	1.4499	.0810	-.3564	-.5569	-.0022	.00000
	11	1.397	7.6	.351	.1279	.0211	.1452	-.0660	-.4547	-.4005	.8808	.7758	.0810	-.2607	-.4612	.0065	.00004
	12	1.403	13.1	.297	.1644	.0216	.1471	-.0776	-.5272	-.5890	1.1173	1.2484	.0821	-.3307	-.5301	.0029	.00001
	13	1.683	21.8	.277	.3026	.0511	.2261	-.1261	-.5575	-.7462	1.3385	1.7916	.1262	-.3962	-.5515	-.0060	.00004
	14	1.403	22.3	.249	.1983	.0216	.1471	-.0888	-.6038	-.8141	1.3483	1.8179	.0821	-.3991	-.5985	-.0053	.00003
	15	2.242	21.8	.320	.4693	.1229	.3506	-.1735	-.4948	-.6623	1.3385	1.7916	.1956	-.3962	-.4821	-.0127	.00016
	16	2.242	32.5	.293	.5301	.1229	.3506	-.1869	-.5331	-.8060	1.5119	2.2858	.1956	-.4475	-.5334	.0003	.00000
	17	2.661	46.8	.299	.7098	.1806	.4250	-.2228	-.5243	-.8757	1.6702	2.7896	.2372	-.4944	-.5387	.0144	.00021
	18	2.239	21.9	.320	.4693	.1226	.3501	-.1732	-.4948	-.6632	1.3404	1.7967	.1954	-.3968	-.4829	-.0119	.00014
	n=17			Sum	5.9326	1.4458	4.6757	-2.2653	-8.3754	-10.4134	20.9446	26.4484			Sum	0.0005	0.00077

a = 0.558
b = -0.296
c = -0.282

Standard deviation = ±0.0067
Probable error = ±0.0045

TABLE VII

FINAL LEAST-SQUARES CORRELATION OF ENGINE-COOLING DATA

FROM TABLE V

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Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test	Run	W_e (lb/sec)	$\sigma_a \Delta p$ (in. water)	$\frac{T_h - T_a}{T_g - T_h}$	xz	x^2	x	xy	y	yz	z	z^2	ax	bz	f(x,z)	δ	δ^2
363	1	2.225	11.0	0.397	0.3617	0.1206	0.3473	-0.1393	-0.4012	-0.4178	1.0414	1.0845	0.1955	-0.3175	-0.3934	-0.0078	0.00006
	2	1.944	10.8	.377	.2983	.0833	.2887	-.1223	-.4237	-.4379	1.0334	1.0679	.1625	-.3151	-.4240	.0003	0
	3	1,669	10.8	.348	.2299	.0495	.2225	-.1020	-.4584	-.4737	1.0334	1.0679	.1252	-.3151	-.4613	.0029	.00001
	4	1.394	11.2	.311	.1514	.0208	.1443	-.0732	-.5072	-.5322	1.0492	1.1008	.0812	-.3199	-.5101	.0029	.00001
	5	2.231	12.4	.386	.3810	.1215	.3485	-.1441	-.4134	-.4520	1.0934	1.1955	.1961	-.3334	-.4087	-.0047	.00002
	6	1.950	20.6	.311	.3810	.0841	.2900	-.1471	-.5072	-.6664	1.3139	1.7263	.1632	-.4006	-.5088	.0016	0
	8	2.233	20.7	.336	.4592	.1217	.3489	-.1653	-.4737	-.6234	1.3160	1.7319	.1964	-.4012	-.4762	.0025	.00001
	9	2.503	20.6	.362	.5236	.1588	.3985	-.1759	-.4413	-.5798	1.3139	1.7268	.2243	-.4006	-.4477	.0064	.00004
	10	1.397	16.0	.276	.1748	.0211	.1452	-.0812	-.5591	-.6732	1.2041	1.4499	.0817	-.3671	-.5568	-.0023	.00001
	11	1.397	7.6	.351	.1279	.0211	.1452	-.0660	-.4547	-.4005	.8808	.7758	.0817	-.2686	-.4583	.0036	.00001
	12	1.403	13.1	.297	.1644	.0216	.1471	-.0776	-.5272	-.5890	1.1173	1.2484	.0828	-.3407	-.5293	.0021	0
	13	1.683	21.8	.277	.3026	.0511	.2261	-.1261	-.5575	-.7462	1.3385	1.7916	.1272	-.4081	-.5523	-.0052	.00003
	14	1.403	22.3	.249	.1983	.0216	.1471	-.0888	-.6038	-.8141	1.3483	1.8179	.0828	-.4111	-.5997	-.0041	.00002
	16	2.242	32.5	.293	.5301	.1229	.3506	-.1869	-.5331	-.8060	1.5119	2.2858	.1973	-.4610	-.5351	.0020	0
	n=14			Sum	4.2842	1.0197	3.5500	-1.6958	-6.8615	-8.2122	16.5955	20.0705			Sum	0.0002	0.00022

a = 0.563
b = -0.305
c = -0.271

Standard deviation = ±0.0040
Probable error = ±0.0027

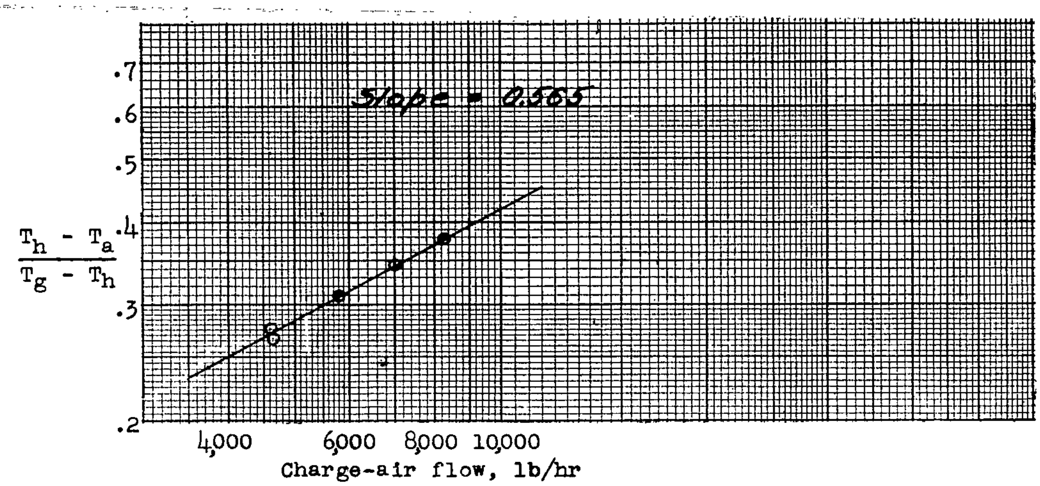

Figure 1.- Construction curve for graphical correlation of cylinder-head temperatures. Fuel-air ratio, 0.08; cooling-air pressure drop, 14.2 inches of water. Data taken from reference 7. (See table I.)

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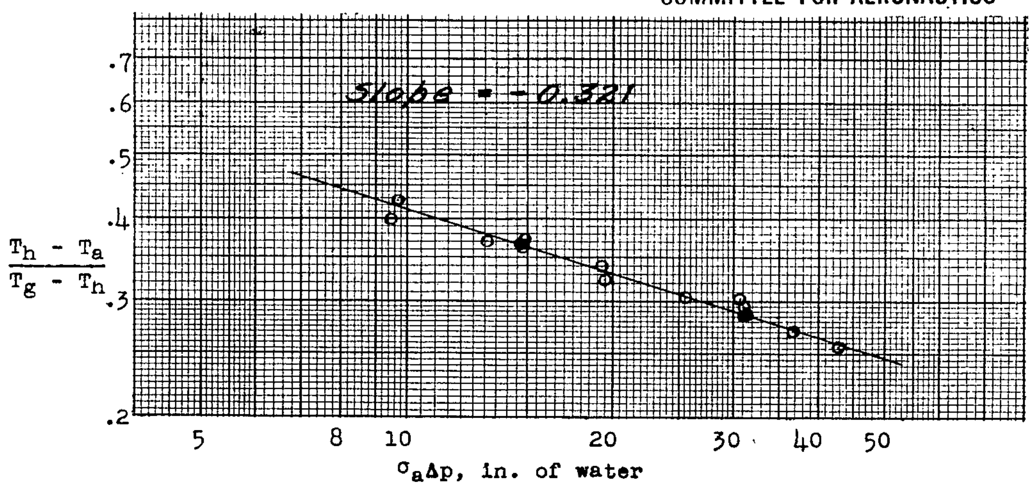

Figure 2.- Construction curve for graphical correlation of cylinder-head temperatures. Fuel-air ratio, 0.08; charge-air flow, 7750 pounds per hour. Data taken from reference 7. (See table I.)

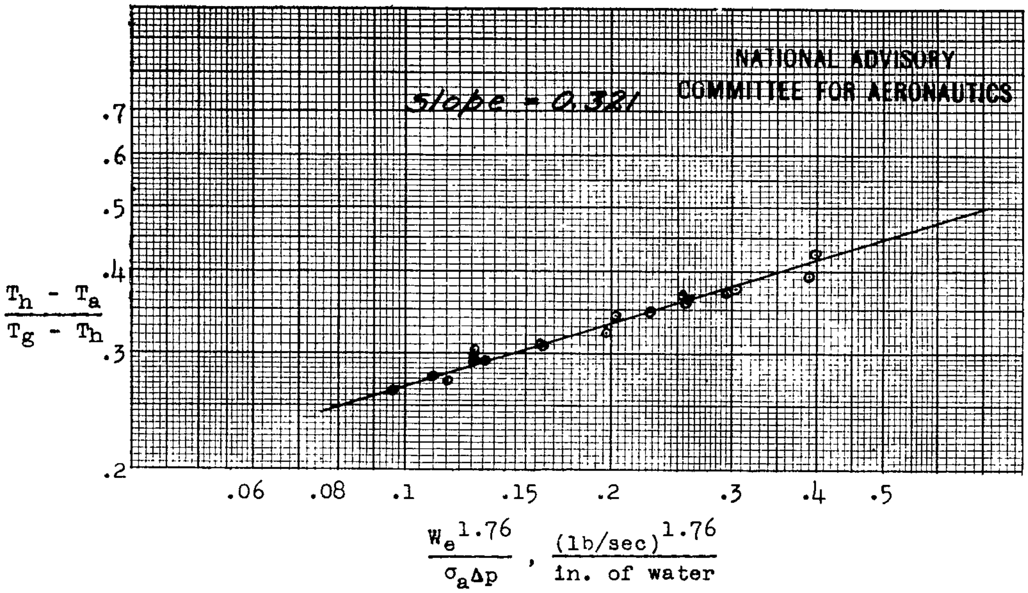

Figure 3.- Graphical correlation of cylinder-head temperatures. P. & W. R-2800 B-series engine; imbedded thermocouples. Data taken from reference 7. (See table I and equation (8).)

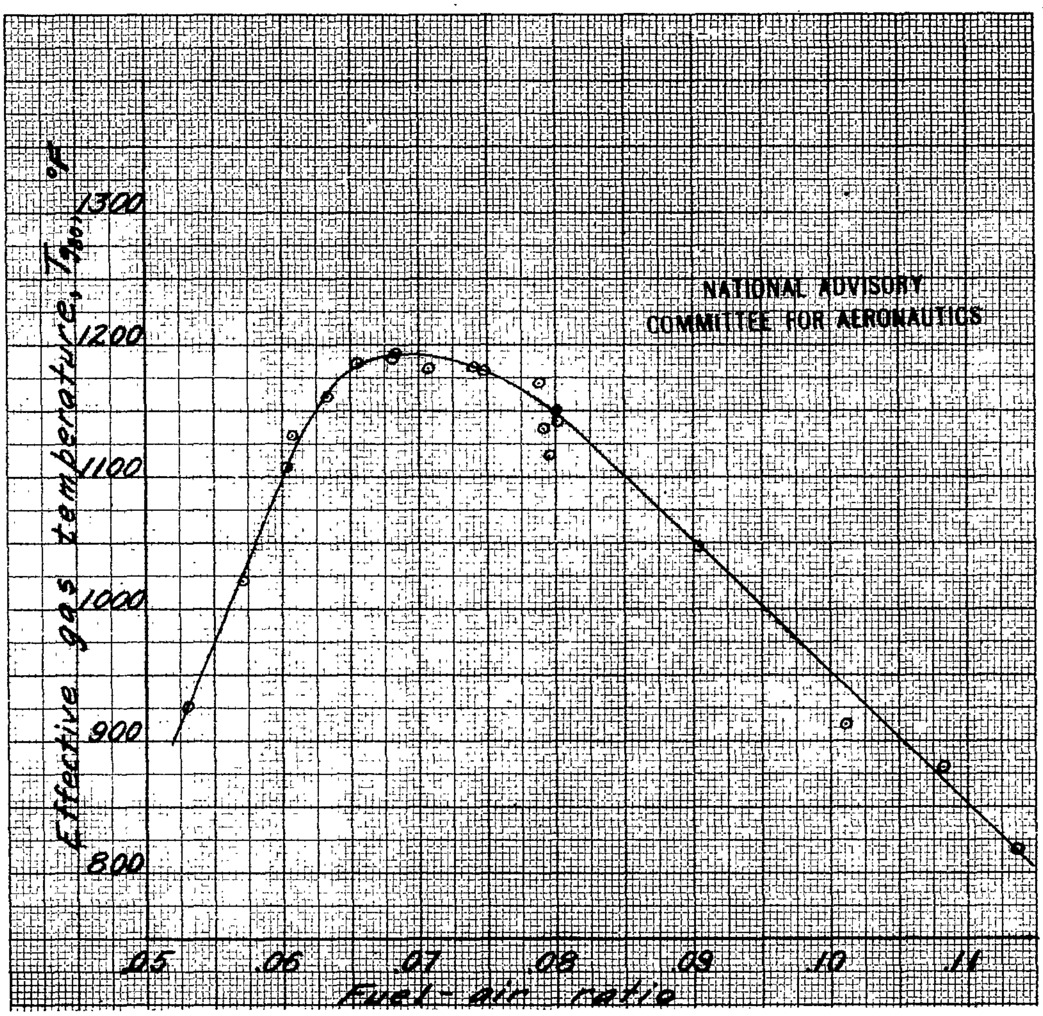

Figure 4.- Variation of reference mean effective gas temperature with fuel-air ratio. P. & W. R-2800 B-series engine cylinder head; nonaromatic fuel. Data taken from reference 7. Use with figures 3 and 6. (See table II and equation (9).)

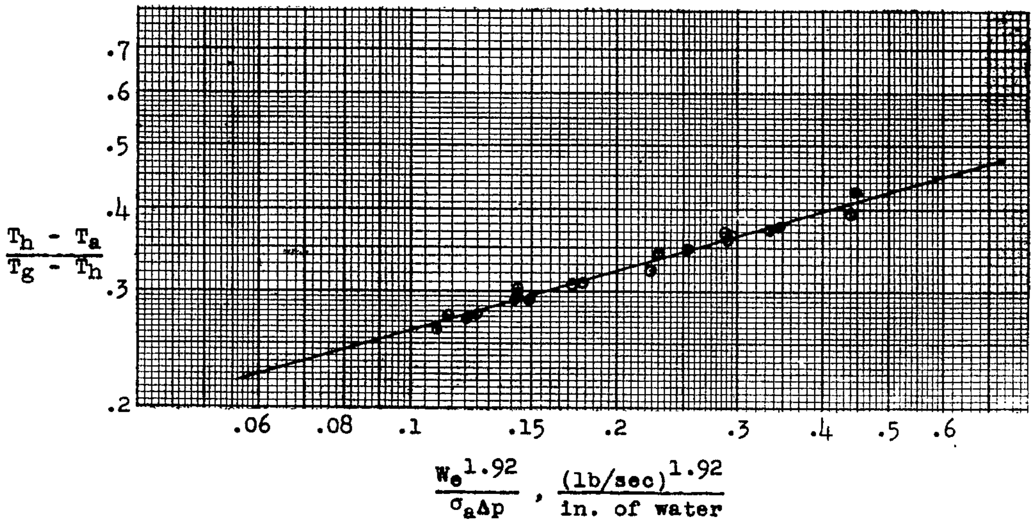

Figure 5.- First least-squares correlation of data that were correlated graphically in figure 3. (See table III and equation (13).)

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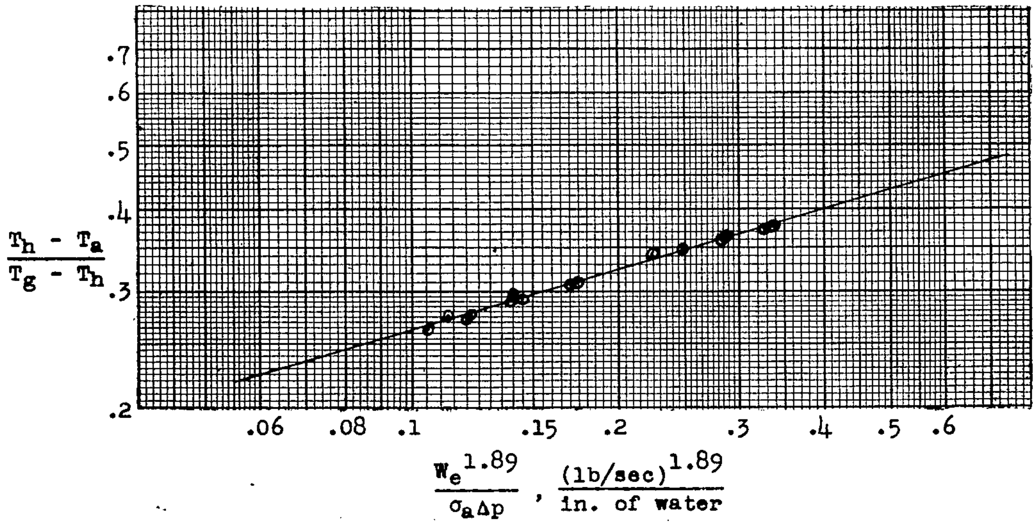

Figure 6.- Final least-squares correlation of select data from figure 3. (See table IV and equation (14).)

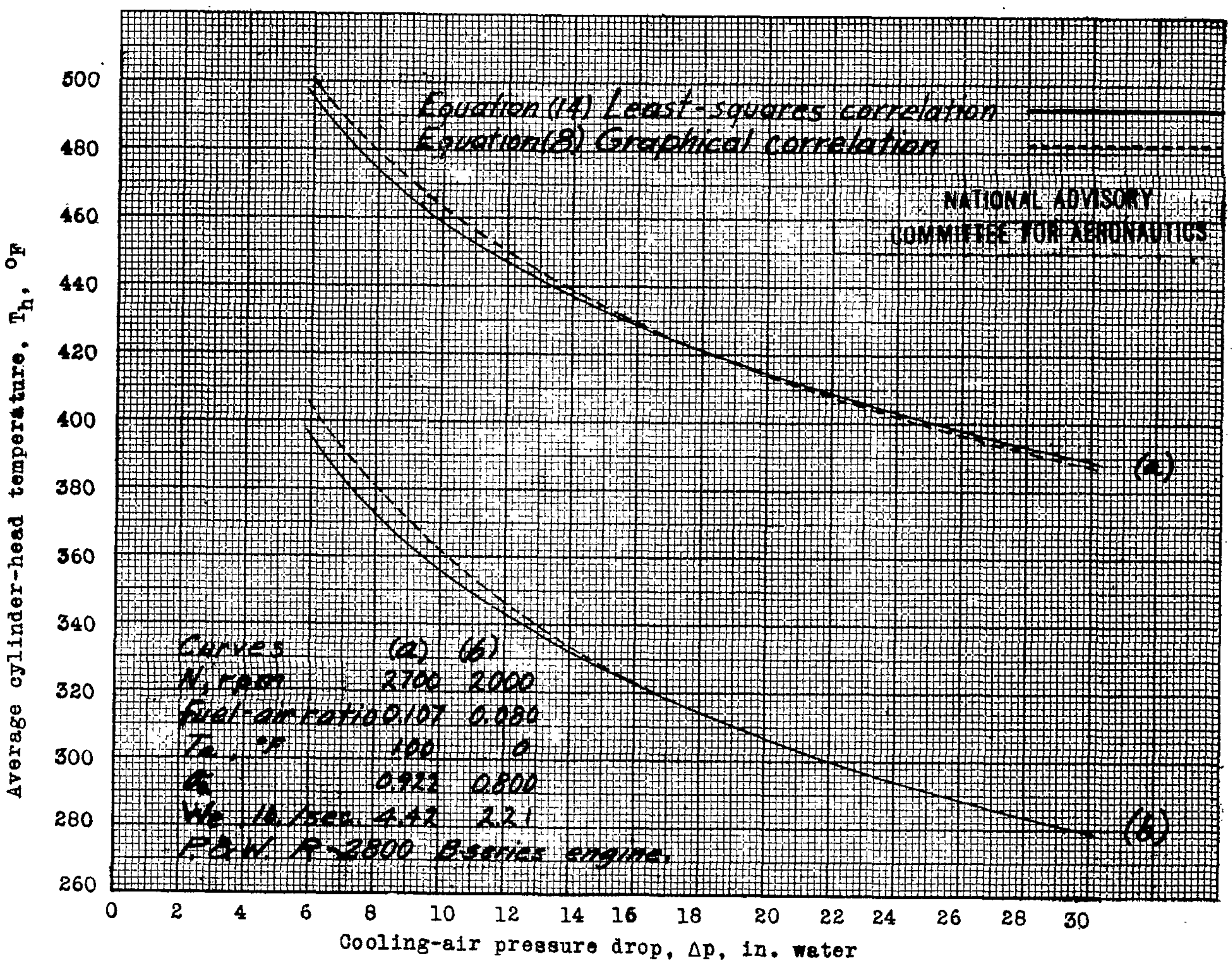

Figure 7.- A comparison of a least-squares correlation with a graphical correlation based on calculated head temperatures.

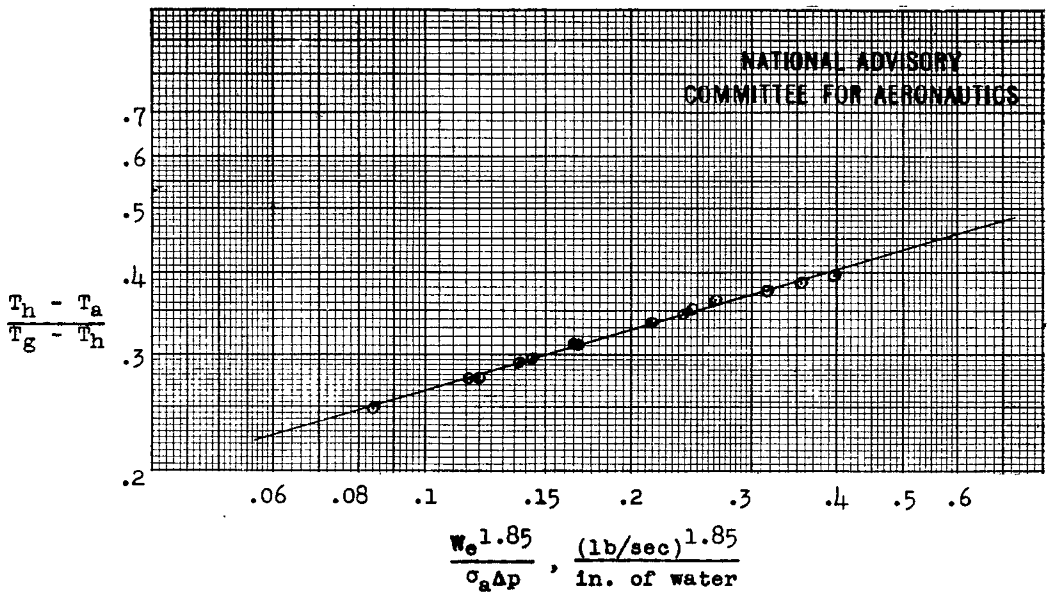

Figure 8.- Final least-squares correlation of miscellaneous data not adapted to graphical correlation. (See table VII and equation (15).)